IJESRT
INTERNATIONAL JOURNAL OF ENGINEERING SCIENCES & RESEARCH TECHNOLOGY

A NOTE ON SPECIAL PAIRS OF PYTHAGOREAN TRIANGLE AND 3-DIGIT SPHENIC NUMBER

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DOI: 10.5281/zenodo.823122

ABSTRACT
In this paper , we present pairs of Pythagorean triangles such in each pair, the difference between their perimeters is four times the 3-digit Sphenic number 110. Also we present the number of pairs of primitive and non- primitive Pythagorean triangles.

KEYWORDS: Pythagorean Triangle, Sphenic number, Prime numbers.

I. INTRODUCTION
Number theory is broad and diverse part of Mathematics that developed from the study of the integers. Mathematics all over the ages have been fascinated by Pythagorean Theorem and problem related to it there by developing Mathematics. Pythagorean triangle which were first studied by the Pythagorean around 400 B.C., remains one of the fascinated topics for those who just adore the number. In this communication, we search for pairs of Pythagorean triangles, such that in each pair, the difference between their perimeters is 4 times the 3digit Sphenic number 110.

II. BASIC DEFINITIONS
Definition 1:

The ternary quadric Diophantine equation given by $x^2 + y^2 = z^2$ is known as Pythagorean equation where x, y, z are natural numbers. The above equation is also referred to as Pythagorean triangle and denote it by T(x ,y, z). Also in Pythagorean triangle T(x,y,z); ix, and y are legs called its and z its hypotenuse.

Definition 2:

The most cited solutions of the Pythagorean equation is $x = m^2 - n^2$, $y = 2mn, z = m^2 + n^2$, where m>n>0. This solution is called primitive, if m,n are of opposite parity and gcd(m,n)=1.

Definition 3:

Sphenic number is a positive integer that is the product of three distinct prime numbers.

III. Main Results
Theorem 3.1:

Let PT_1, PT_2 be two distinct Pythagorean triangles with generators $m, q(\, m > q > 0)$ and $p, q(\, p > q > 0)$ respectively. Let P_1,P_2 be the perimeters of PT_1,& PT_2

Such that $P_1 - P_2$ = 4times the 3-digit Sphenic number $\frac{P_1 - P_2}{4}$.

The above relation leads to the equation

$$(2m+q)^2 - (2p+q)^2 = 880 \qquad (1)$$

Which simplifies to

$$(m-p)(m+p+q) = 220 \qquad (2)$$

After completing the numerical computations, it is noted that there are 56 values of m, p & q

ISSN: 2277-9655
Impact Factor: 4.116
CODEN: IJESS7

Satisfying Equation (1).

We have presented the values of m, p, q ,P_1, P_2 in the following table

S. No	m	p	q	P_1	P_2	$\frac{P_1 - P_2}{4}$
1	110	109	1	24420	23980	110
2	109	108	3	24416	23976	110
3	108	107	5	24408	23968	110
4	107	106	7	24396	23956	110
5	106	105	9	24380	23940	110
6	105	104	11	24360	23920	110
7	104	103	13	24336	23896	110
8	103	102	15	24308	23868	110
9	102	101	17	24276	23836	110
10	101	100	19	24240	23800	110
11	100	99	21	24200	23760	110
12	99	98	23	24156	23716	110
13	98	97	25	24108	23668	110
14	97	96	27	24056	23616	110
15	96	95	29	24000	23560	110
16	95	94	31	23940	23500	110
17	94	93	33	23876	23436	110
18	93	92	35	23808	23368	110
19	92	91	37	23736	23296	110
20	91	90	39	23660	23220	110
21	90	89	41	23580	23140	110

22	89	88	43	23496	23056	110
23	88	87	45	23408	22968	110
24	87	86	47	23316	22876	110
25	86	85	49	23220	22780	110
26	85	84	51	23120	22680	110
27	84	83	53	23016	22576	110
28	83	82	55	22908	22468	110
29	82	81	57	22796	22356	110
30	81	80	59	22680	22240	110
31	80	79	61	22560	22120	110
32	79	78	63	22436	21996	110
33	78	77	65	22308	21868	110
34	77	76	67	22176	21736	110
35	76	75	69	22040	21600	110
36	75	74	71	21900	21460	110
37	55	53	2	6270	5830	110
38	54	52	4	6264	5824	110
39	53	51	6	6254	5814	110
40	52	50	8	6240	5800	110
41	51	49	10	6222	5782	110
42	50	48	12	6200	5760	110
43	49	47	14	6174	5734	110
44	48	46	16	6144	5704	110

45	47	45	18	6110	5670	110
46	46	44	20	6072	5632	110
47	45	43	22	6030	5590	110
48	44	42	24	5984	5544	110
49	43	41	26	5934	5494	110
50	42	40	28	5880	5440	110
51	41	39	30	5822	5382	110
52	40	38	32	5760	5320	110
53	39	37	34	5694	5254	110
54	23	18	3	1196	756	110
55	22	17	5	1188	748	110
56	21	16	7	1176	736	110
57	20	15	9	1160	720	110
58	19	14	11	1140	700	110

Thus it is seen that there are 58 pairs of Pythagorean triangle such that in each pair the difference between the perimeters is 4 times the 3-digit Sphenic number 110.

Out of 58 pairs, there are 38 pairs of primitive Pythagorean triangles, 8 pairs of non-primitive Pythagorean triangles and remaining 12 pairs, one is primitive and other is non-primitive.

IV. CONCLUSION

In this paper, it is observed that there are only finitely many Pythagorean Triangles satisfying the property under consideration. The total Pythagorean triangles are number of pairs of primitive and non-primitive Pythagorean triangle are also given. To conclude, one may search for the connection between the pairs of Pythagorean Triangle and other Sphenic number of higher order.

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Mohamed , S. Y., & Kumar, M. P. (2017). A NOTE ON SPECIAL PAIRS OF PYTHAGOREAN TRIANGLE AND 3-DIGIT SPHENIC NUMBER. *INTERNATIONAL JOURNAL OF ENGINEERING SCIENCES & RESEARCH TECHNOLOGY, 6*(7), 242-246. doi:10.5281/zenodo.823122